Money, Credit and Central Banks. Heretic Thoughts.*

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Abstract

I build a dynamic general equilibrium model aimed at making sense of the so-called "Austrian Theory of Money and Credit". Results are mixed but, I hope, informative at least from a historical perspective. Next, I ask the model if it is true that "low" interest rates always cause booms and busts, as it is often claimed. Also in this case, the verdict is interestingly ambiguous. It is volatile, more than low, interest rates that may, under certain circumstances, give rise to sequences of booms and busts. This finding has both theoretical and practical implications supporting the old view that rules, in central banking, are in general preferable to discretion.

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1. Introduction

This is a preliminary and higly incomplete paper. It should not be circulated for any purpose other than that of this seminar.

I make an attempt at formulating a dynamic general equilibrium version of what is generally labeled as "Austrian Monetary Theory" or "Austrian Theory of the Business Cycle". Such theories are generally associated to the names of Ludwig von Mises and Friedrich Hayek (among other, more detailed references should be added) and have been somewhat out of fashion for a while in standard macroeconomics. Part of the reason for their falling out of fashion appears to be their "vagueness", i.e. the fact that most of their intuitions have never been formalized, and arguments are never proven rigorously but only by means of very special examples. Here I make an attempt to recast what I believe to be a reasonable version of the "Austrian Monetary Theory" in a setting with which modern macroeconomists should be familiar enough. I then use the model to derive the aggregate implications of dieeren3t sets of intitutional asumpti15(o)11(n)11(s) marized onl68(y)-328(i)6(n)12(f)6(o)10(r)11(m)18(a)10(l)6(l)6(y)-328(m)17(a)11(k)11(i)6(n)11(g)-328(t) proof71(s)-319(w)15(i)6(l)6(l)-321(e)10(v)38(e)9(n)39(t)8(u)11(a)11(l)6(l)5(y)-315(b)-16(e)-317(a)11dud

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oonet71snel68(i73(e)10vr)10(e)-252(t78(h)11(a)11(t)-270(a)-240(s)9(u)12(s)8(t)8(a)19(i)6(n)32(e610(d)-24 *the theory, the business cycle unfolds in the following way: Low interest rates tend to stimulate borrowing from the banking system. This expansion of credit causes an expansion of the supply of money, through the money creation process in a fractional reserve banking system. It is asserted that this leads to an unsustainable credit-sourced boom during which the artificially stimulated borrowing seeks out diminishing investment opportunities. Though disputed, proponents hold that a credit-sourced boom results in widespread malinvestments. In the theory, a correction or "credit crunch" – commonly called a "recession" or "bust" – occurs when exponential credit creation cannot be sustained. Then the money supply suddenly and sharply contracts when markets finally "clear", causing resources to be reallocated back towards more efficient uses.*

Given these perceived damaging and disruptive effects caused by what Austrian scholars believe to be volatile and unsustainable growth in credit-sourced money, many proponents (such as Murray Rothbard) advocate either heavy regulation of the banking system (strictly enforcing a policy of full reserves on the banks) or, more often, free banking.[3] The main proponents of the Austrian business cycle theory historically were Ludwig von Mises and Friedrich Hayek. Hayek won a Nobel Prize in economics in 1974 (shared with Gunnar Myrdal) in part for his work on this theory.[4][5]

The Austrian explanation of the business cycle varies significantly from the mainstream understanding of business cycles, and is generally rejected by mainstream economists.

Because, on the one hand, I am not sure I have fully understood what the Austrians mean in many of their writings and, on the other hand, I have also found some of their statements dubious at least, the (set of) models I present below, while inspired by some Austrian writers, are most certainly "pseudo-Austrian". While the notes contained here are part of a more ambitious project, here a brief list of the things you will find in the following pages.

1. I develop a dynamic general equilibrium model of a credit economy with production and flexible prices, in which the Austrian ideas about the origin of credit, fractional reserves, fiat money and Central Bank's (CB, from now on) role can be analyzed.

2. Under private banking (absent a CB) the economy has a competitive equilibrium where a "natural real rate of return" (reminiscent of Knut Wicksell's concept) obtains. Other than for external shock, this economy tends to live near or at its steady state.

3. Contrary to a long-held Austrian view, the competitive equilibrium with only private banking - and, either, no fractional reserve, or finite fractional reserve and no fiat money, or fractional reserve and fiat money and a strictly positive interest rate on money - is not optimal.

4. The allocations described in 3 can be improved upon by the introduction of, respectively, a fractional reserve system, fiat money, a CB and, in fact, a monetary policy setting the nominal interest rate at zero. In a particular, but relevant, case the usual Friedman Rule does apply and it implements the first best.

5. CB's policies may have real effects, in the sense that the CB may or may not implement the first best and this leads to different real allocations.

6. Once "full employment" is reached, monetary policy has no further real effects and a more "loose" policy only changes the price level.

7. When monetary policy has real effects, these include an increase in the value of risky assets.

8. In a subset of these circumstances, monetary policy leads to an increase in the nominal value of assets without changes in the price of output.

9. Sustained credit expansion that are suddenly reversed may lead to oscillations resembling booms and busts, at least in a stylized sense.

10. Still, a boom-bust obtains only when the CB makes mistakes, either by using the wrong model of the economy or by incorrectly interpreting the signals it receives from the private economy.

The rest of the (incomplete) paper (and of my presentation) proceeds as follows. In Section 2, I study a sequence of constrained central planner problems that are meant to clarify what is it that banks and fiat money do in my economic environment. In section 3 I introduce the basic elements of the decentralized economy. In section 4, I list the main properties of the competitive equilibrium. In section 5 I extend the model to allow for the simultaneous presence of "short" and "long" production processes, and derive the conditions under which a boom-bust obtains in equilibrium. All the rest is missing: proofs, calibration, analysis of the data, simulations, references and a better and more interesting introduction.

2. Three Central Planner Problems

I start by describing three aggregate economies in which a fictitious and benevolent (how could it be otherwise?) Central Planner (CP) determines the intertemporal allocation under different institutional constraints that are induced (through an act of the CP) by different assumptions about the relevant informational and contractual frictions. We will, eventually, identify such different institutional constraints with the financial regimes briefly mentioned in the introduction.

This section should serve as a synthetic illustration of my view - which should probably be labelled as "eventually Panglossian" - about the functional roles and historical origins of banking, bank notes, fractional reserves, fiat money and central banks. A more detailed discussion of such historical and theoretical issues are in Boldrin [2015].

2.1. The Frictionless Economy

Our starting point is a particular version of the classical Brock&Mirman [1972] economy with two aggregate production functions, a representative household and technological uncertainty.

At the start of period t, the economy is endowed with two kinds of capital stock: k_t and η_t^0. The stock k_t is irreversibly invested in a risky technology, while η_t^0 consists of stored output that can be either consumed or invested (η_t) in a safe production process. Labor time, L, is also available that may be used (to keep things simple) only in the risky technology. The technological and resource constraints are:

$$c_t + \eta'_{t+1} + k_{t+1} = Y_t = z_t F(k_t, \ell_t) + (1 + r)\eta_t,$$
$$\eta_t \leq \eta'_t$$
$$\ell_t \leq L.$$

Aggregate output, Y_t, is the sum of what obtains from the risky, $Y_t^1 = z_t F(k_t, \ell_t) = z_t G(k_t, \ell_t) + (1 - \mu)k_t$, and the riskless, $Y_t^2 = (1 + r)\eta_t$, technologies. The function $G(k_t, \ell_t)$ is a strictly-decreasing returns to scale production function of two inputs (an entrepreneurial ..xed factor suppliers work on the basis of a belief that their credit will be honored. In other words, they really no "credit", hence no banks of any kind. What this observation makes clear, though, is that even if we had money, either "flat" or commodity money, to be used in transactions, there would still be room for some other social compact that could improve the equilibrium allocation. Technically speaking, this means that a transaction demand for money, of the kind arising for example in the class of models that follow on the pathbreaking work of Kiyotaki and Wright [1989], could be appended to the constrained Brock and Mirman model I have sketched here, without altering the conclusions reached so far, and those to follow as well.

2.3. Small Friction: Private Banking

Assume, next, that the safe production process is fully observable, hence workers can verify both what is invested there and what the rate of return is. Furthermore, the safe production process takes place under everyone's eyes, and its output cannot be hidden away; implying that promises to pay part of it to someone at the end of the period are costless to enforce. This enables the CP to make workers a credible promise of future payment by handing them IOUs payable in units of the output of the safe technology. To the extent the workers can monitor the riskless process, and collect their payments from there in case something goes wrong with the risky technology, this improves upon the allocations attainable in the trust-less economy.

This does not take us back to the first-best world, though, insofar as the amount of "labor" that can be employed in the concave technology is constrained by the amount of resources previously stored away in the safe technology. In other words, the IOUs that the CP issues (or that, in a decentralized version, the private banks having access to the safe-and-show technology would issue) must be "fully backed" by some asset or commodity. In the private banking case we can think of this as an economy with a 100% reserve requirement ratio, as advocated, among other, by Mises and his followers.

The "credit constraint" facing the CP is

$$v'(L-\ell_t)\ell_t \leq u'(c_t)(1+r)\eta'_t,$$

hence the first order condition determining the level of employment reads

$$v'(L-\ell_t) \leq z_t F_2\left(k_t, \ell_t\right) u'(c_t)$$

with equality when the previous constraint is not binding.

When the positive shock, $\bar{z}_t$, is large enough the credit constraint may be binding while the first order condition may still show a strict inequality, leading to inefficiently low employment and output levels. When this event is likely, the intertemporal choice is distorted as the CP - to reduce the probability of the credit constraint binding next period - will save more in the storage technology, i.e. in η'_t, than otherwise efficient.

In summary: while fully-backed private banking is a useful tool that substantially improves over the equilibrium without any credit, there are still efficiency gains from introducing flat money via a Central Bank, which we consider next. The extension to private banking under fractional reserves is straightforward and leads to similar conclusions, hence I will omit it here.

2.4. An Economy with Fiat Money

Next we assume that "fiat money" is available: the planner (behaving as a CB) can decide how much of it to issue and at what price to "lend" it to the private banks. The money issued by the CB is "fiat" in the sense that is unbacked by nothing else but the expropriating power of the state, which controls the CB and can appropriate future output (possibly at some positive social cost). Again, for the sake of brevity, I will omit here the details of the, rather simple, micro model supporting such behavior (references TBA).

This "institution" amounts to assuming the Central Planner can credibly promise that future payments will be obtained from the output of the risky technology. After the aggregate shock is revealed, the CP hires workers by issuing IOUs on Y^1, which they accept in exchange of their labor effort. They are redeemed, at end of period, in exchange for a portion of Y^1 and start by assuming there is only one souce of uncertainty, the aggregate productivity shock. It a¤ects the risky investment projects by altering the size of their payo¤s in the successful state; safe investment projects are also available. Apart from the aggregate shock, risky investment projects are irreversible for one full period while safe investments are not, as detailed below. Central Bank's policies a¤ect individual agents by altering the conditions under which they can borrow and lend safely. In the basic model we treat those policies as deterministic; a stochastic policy function for the Central Bank will be introduced later on.

Households maximize their discounted utility from consumption and leisure over an in…nite horizon. Their earnings consist of both labor and capital income. The latter consists of (a) banks' pro…ts, (b) interest on bank's deposits, and, (c) capital gains on shares of the …rms carrying out the risky projects. Portfolio's allocations take place at the end of each period, under uncertainty about tomorrow's state of the world, while transactions involving "money" that goes in and comes out of bank accounts and loans take place also during the period.

Firms last one period, own the capital stock, and are owned by the households; they may borrow from banks on a period-by-period basis. Firms carry out the risky investments, while only banks have access to the safe ones. Banks collect interest-bearing deposits from households, borrow/lend funds from/to the Central Bank (CB), invest in the safe technology, and lend to …rms.

The Central Bank (CB) controls the rate of interest on short-run funds (r_t), and the maximum leverage ratio (θ_t) it allows banks to establish between the commercial loans (B_t) they create and time deposits (D_t) they keep invested in the safe assets. The CB stands ready to o¤er any amount of loanable funds the banks demand (i.e. to "create liquidity") at the rate r_t, until the leverage ratio of the private banking sector reaches θ_t.

During each period, markets open in the following sequence:

- *corporate bonds market*, loans from banks to …rms are traded;
- *demand deposits market*, short run deposits from households into banks;
- *labor market*, labor is hired by …rms;
- *output market*, aggregate output is traded to be either consumed or invested;
- *stock market*, shares of …rms are traded;
- *time deposits market*, deposits from the households to the bank are traded.

More in details. When a period starts …rms are endowed with equity capital, sunk until after production is completed and banks have their deposits invested either in the safe technology or in reserves at the CB; both are reversible on call. Uncertainty resolves: the aggregate shock is realized and the CB announces its monetary policy stance. To carry out production, …rms need to purchase "labor" from households on the spot market. As payments, households accept either output or "money", which is issued by the CB; either of them …rms must borrow from banks. Banks can either lend part of last period's output, removing it from the safe technology where it is invested, or "money" borrowed from the CB; whichever is more convenient to them. After labor is hired production is carried out, factors payment take place and output becomes available. Part is consumed, part invested in the equity capital of …rms operating next period, part is deposited with the banks, at which point the period ends.

This being a monetary economy, we use nominal prices expressed in the unit of account called "money", which is issued by the CB.

3.1. Households

The representative household's labor income is $w_t\ell_t$, where w_t is the nominal wage rate and $\ell_t \leq L$ the labor supply. In the baseline case we set, $v(L-\ell_t) = v\cdot(L-\ell_t)$, which is useful to stress the fact that "labor", here, is just a useful stand-in for any input other than those acquired *ab-initio* through equities. Capital income has three sources: shareholdings (s_t), bank deposits (d_t), and bank profits (φ_t). Total income (wealth, in fact) is used to purchase consumption goods, c_t, new shares, s_{t+1}, and new bank deposits, d_{t+1}, as ownership of the bank is perpetual. Households solve:

$$\max_{\{c_t, s_{t+1}, d_{t+1}, \ell_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \delta^t [u(c_t) + v(L-\ell_t)] \tag{3.1}$$

subject to: (3.2)

$$p_t c_t + \int_0^1 q_t(\alpha)s_{t+1}(\alpha)d\alpha + d_{t+1} = w_t\ell_t + (1+i_t)d_t + \int_0^1 n_t(\alpha)s_t(\alpha)d\alpha + \varphi_t \tag{3.3}$$

$c_t, \ell_t, s_{t+1}, d_{t+1} \geq 0$, plus transversality.

Here and in what follows, p_t is the price of output; $q_t(\alpha)$ the price at which a share in firm α, operating in period $t+1$, may be purchased at the end of period t; $n_t(\alpha)$ the market value of a share in firm α at the end of period t; i_t the interest paid by banks on d_t;φ_t the banks' profits, accruing to households. All these quantities, but the interest rate that is a pure number, are expressed in current units of account. The utility functions u and v are concave and satisfy standard conditions.

Within each period, the timing for households is as follows: after the aggregate uncertainty is lifted they sell labor to firms for a payment of $w_t\ell_t$, which they deposit in the bank, production is then carried out and households receive capital income from the firms and the banks, at which point consumption and saving take place.

3.2. Technology and firms

A risk-free technology is available, to which only banks have access. Output can be stored there at the end of each period. When the new period starts the stored output can either be recovered and used somewhere else, or left invested in the safe technology, yielding a total real return of $1+r>0$ by the end of the same period. We use $\eta_t \leq \eta_t'$ to denote the physical amount of resources finally retained for production, whereas η_t' is the amount stored at the end of the previous period $t-1$. Hence, output from the safe techonology is

$$Y_t^2 = (1+r)\eta_t.$$

In each period, different types of risky investment projects are also available that we index by their probability of success, $\alpha \in [0,1]$. To make our life easier, the measure of projects of type α, each involving a unit-size investment, is $\mu(\alpha) = 1/\alpha$ - in Appendix I (TBA) we work out the details of this and of the more general exponential case. Given the aggregate productivity shock, $z_t \in \{\underline{z}_t, \overline{z}_t\}$, the gross productivity of an investment project of type α is

$$z_t(1+a), \text{ with probability } 0 \leq \alpha \leq 1;$$

$$0, \text{ with probability } 1-\alpha.$$

Write $A_t(\alpha) = \alpha z_t(1+a)$ to denote the expected return on a project of type α. To save on notation, the dependence of A_t on α will be omitted; also, we use $\underline{A}_t$ and $\overline{A}_t$ to denote the payo¤ associated with, respectively, the low and the high aggregate shock. If $0 < k(\alpha) \leq 1/\alpha$ units of capital are invested in projects of type α, $k(\alpha)$ independent projects of that type will be implemented. Assume each project has a Cobb-Douglas production function of "all inputs other than capital" that, to simplify, we summarize here with just "labor". Total output from each family α of projects will therefore be

$$y_t(\alpha) = \int_0^{k_t(\alpha)} A_t(\alpha)\ell_t(\alpha)^{1-\#} ds = A_t(\alpha)k_t(\alpha)\ell_t(\alpha)^{1-\#},$$

and aggregate output from the risky technology is

$$Y_t^1 = \int y_t(\alpha) d\alpha.$$

We identify each class of equities with a class of investment projects: projects of type α are exclusive to "…rm" α[2]. At the end of each period, households invest in …rms by purchasing their shares at a unit price of $q_{t-1}(\alpha)$; we normalize at one the number of shares of each …rm, hence $q_{t-1}(\alpha)$ is the market value of the …rms of class α at the end of period $t-1$ and at the beginning of t. At the start of the period, …rms observe the aggregate productivity shock, z_t, and the monetary policy stance, r_t and θ_t; then they hire labor and carry out their production projects, the output of which becomes available at the end of the period. The hiring of labor needed to complete the production process are …nanced through bank loans, the nominal amount of which we denote with $b_t(\alpha)$.

The initial shareholders' capitalization endows the …rm with $k_t(\alpha) = q_{t-1}(\alpha)/p_{t-1}$ units of productive capacity, which is sunk when period t begins. Therefore, before uncertainty is resolved, the beginning-of-period market value of …rms is

$$V_t = \int q_{t-1}(\alpha) d\alpha.$$

Bank loans, $b_t(\alpha)$, are used to purchase "labor", a stand-in for all other inputs in the baseline model. Households are not willing to lend their "labor" to the …rms - maybe because of a private information problem, or maybe because the length of the production "period" is long relative to their consumption needs, or just because ... - and ask for payment up front. Firms must borrow from banks to …nance such payments. With a a loan equal to $b_t(\alpha)$, $\ell_t(\alpha) = b_t(\alpha)/w_t$

[2] The careful reader will notice an ambiguity that borders "handwaving" here ... a …rm or a sector or a "class of equities"? Indeed, the industrial organization portion of this model still needs ironing!

units of "labor" are acquired and productive capacity then becomes

$$y_t(\alpha) = \int_0^{k_t(\alpha)} A_t(\alpha)\ell_t(\alpha)^{1-\#} = \int_0^{q_{t-1}(\alpha)/p_{t-1}} A_t(\alpha)\left[b_t(\alpha)/w_t\right]^{1-\#} ds.$$

Let i_t^b be the nominal interest rate on loans. The end-of-period market value of firms of type α is

$$n_t(\alpha) = p_t y_t(\alpha) - (1+i_t^b)b_t(\alpha) = \frac{p_t}{p_{t-1}} A_t(\alpha)\left[b_t(\alpha)/w_t\right]^{1-\#} q_{t-1}(\alpha) - (1+i_t^b)b_t(\alpha).$$

In other words, the rate of return on shareholders' investment is

$$\xi_t(\alpha) = \frac{n_t(\alpha)}{q_{t-1}(\alpha)} = \pi_t A_t(\alpha)\left[b_t(\alpha)/w_t\right]^{1-\#} - (1+i_t^b)\theta_t(\alpha),$$

where $\pi_t = p_t/p_{t-1}$ is the rate of inflation, and $\theta_t(\alpha) = b_t(\alpha)/q_{t-1}(\alpha)$ is the leverage ratio of the firm.

To determine firms' demand for loans, proceed as follows. When periods t begins firm α has capitalization $q_{t-1}(\alpha)$; note that $q_{t-1}(\alpha) = 0$ is possible, and likely to be true for values of α near 0. Upon observing z_t, i_t^b and the maximum admissible leverage level $\overline{\theta}_t(\alpha)$, firms proceed to maximize profits by borrowing according to

$$b_t(\alpha) = \min\left\{\frac{(1-\vartheta)q_{t-1}(\alpha)}{(1+i_t^b)w_t^{1-\#}}\pi_t A_t(\alpha),\ \overline{\theta}_t(\alpha)q_{t-1}(\alpha)\right\}.$$

As expected, high initial market valuation, high inflation rates and high productivity shocks lead to higher levels of nominal borrowing. These, in turn, lead to higher level of real activity, higher "wages", and higher end-of-period market value of firms.

Notice that the credit mechanism studied here is different from the one adopted in models of the "financial accelerator" variety (e.g. Bernanke and Gertler, or Kiyotaki and Moore, exact references TBA), with which it is nevertheless compatible and, in fact, complementary. In particular, in a more general model, one can allow for both "unsecured" (the one modeled here) and "secured" or "collateralized" credit. An increase in the amount of unsecured credit available leads to an increase in the market valuation of firms, which may in turn lead to an increase of collateralized credit, and so on, amplifying the financial accelerator mechanism and endogenizing it. In turn, a reduction in the amount of unsecured credit available (due, e.g., to an increase in the interest rate banks charge for it) leads to a drop in the market value of firms and hence, through the financial accelerator that collateralized credit implies, to a further reduction of credit and of economic activity. To put it differently, the type of credit I study is a possible channel through which the exogenous shocks reducing the value of the collateral, and setting in motion the financial accelerator mechanism, could be reasonably endogenized.

3.3. Banks

There is a continuum of identical banks or, which is the same, a price taking representative bank. Households own banks through an untraded perpetual share, while their interest-bearing deposits act as the bank's working capital. This is a limitation of the model, and moral hazard's considerations for the bank sectors - arising from the distinction between own capital and deposits - will have to be addressed elsewhere.

With the deposits received at the end of period $t-1$, banks acquire $\eta_t^0 = d_t/p_{t-1}$ units of period $t-1$ output that will yield a gross return of $1+r$ next period, if invested in the safe technology. In the meanwhile, the output is stored and kept "overnight". Because the banking sector is competitive, the rate of interest promised to depositors will equal r plus expected inflation, i.e. $i_t = r + \pi_t^e$. Because the investment in the risk-free technology is reversible, this is a safe way to store funds between the end of one period and the beginning of the next, waiting for aggregate uncertainty to resolve.

While households commit their deposits at the end of the previous period, the banks' portfolio allocation is determined only after uncertainty is realized. When period t starts, banks hold those deposits either as reserves with the CB or invested in the safe assets. Their balance sheet looks like this

Balance Sheet at starting of period t

Safe Assets $p_t\eta_t^0$	Deposits from Households d_t
Reserves with CB R_t^0	Equity e_t

Where the (positive or negative) value of equity is given by $e_t = (p_t - p_{t-1})\eta_t^0$ and is, at this point of our story, purely virtual (pretend, to simplify, the bank starts period t with zero reserves).

Next comes the CB intervention: this amounts to announcing a policy stance, i.e. a pair of values (r_t,θ_t) and carrying out the open market operations needed to achieve the target rate. An open market operation, in this model, consists in buying/selling safe assets from/to the representative banks in exchange for cash. At this stage, we can think of the latter in the form of reserves held at the CB: its open market operations change the composition of banks portfolios at the beginning of the period. Hence, the balance sheet of the representative banks looks like

Balance Sheet after open market operation

Safe Assets $p_t\eta_t$	Deposits from Households d_t
Reserves with CB R_t	Equity e_t

Let γ be the reserve requirement, which we assume constant at this stage. Banks hold reserves with the CB and, during period t, they may borrow from the CB's discount window at a rate r_t in order to lend to firms. Write $D_t = p_t\eta_t$ and denote with B_t the amount they lend. Recall it should satisfy

$$B_t \leq \min\{\theta_t D_t; \gamma R_t\}.$$

Assume the loans issued by banks are also completely deposited within the banking system (i.e., all commercial transactions are being mediated through bank notes). After all the borrowing and lending is completed, and before interests are paid, the banks' balance sheet looks as follows:

Bank's Assets and Liabilities

Safe Assets D_t	Deposits from Households d_t
Loans to firms B_t	Demand deposits m_t
Reserves with CB R_t	Own Equity/Profits e_t

Consider bank's actions in more details. At the end of period $t-1$ banks receive d_t units of money from households, to whom they promise to pay back $(1+i_t)d_t$ units of money next period. Banks use the d_t units of money to purchase $\eta_t^0 = d_t/p_{t-1}$ units of periot $t-1$ output to be held "over night" in the safe technology. Those η_t^0 units are worth $\eta_t^0 p_t$ units of money the morning after. Of these η_t^0 units, the bank keeps $\eta_t \leq \eta_t^0$ invested in the safe technology, for a nominal value of $D_t = \eta_t p_t$, while the remaining is kept in the form of reserves R_t with the CB. Out of these reserves the bank creates risky loans by issuing credits to the …rms, in the amount B_t, which is constrained twice. Once, operationally, it is constrained by the fractional reserve ratio $\gamma > 0$, which we assume …xed at this time[3]; secondly it is constrained by the leverage ratio θ_t the CB sets between the risky and the safe portions of the bank's balance sheet. Notice that, because of our simplifying assumption that there is no transaction demand for money, the nominal amount of demand deposits, m_t, held with the banking systems (by the workers, as we will see) is equal to the amount lent to …rms, B_t.

Recall also that those η_t units kept invested in the safe technology (as a result of, both, the CB open market operation and banks' portfolio allocation choices) yield a total output of $Y_t^2 = (1+r)\eta_t$ units at the end of period t, with a nominal value of $(1+r)\eta_t p_t = (1+r)D_t$.

Summing up: given a monetary policy stance, the optimization problem of a bank is

$$\max_{B_t;D_t;R_t} \varphi_t = (1+i_t^b)B_t + (1+r)D_t + (1+r_t)R_t - (1+i_t)m_t - (1+i_t)d_t,$$

$$\text{s.to: } B_t + D_t + R_t \leq d_t + m_t + e_t$$
$$B_t \leq \min\{\theta_t D_t; \gamma R_t\}$$

A few things should be noted. Because bygones are bygones, capital gains or losses on the safe assets, induced by the open market operations of the CB, are already sunk at the time the bank makes its lending decisions. Hence, as long as $i_t^b \geq r/\theta_t$ and the reserve ratio is not binding, banks will want to increase B_t until it equals $\theta_t D_t$ and pay no interest on demand deposits m_t (i.e. $i_t = 0$). Perfect competition then implies that $i_t^b = r_t$ will hold and that the lower bound on the (real) short term rate the CB can set is given by $\underline{r}_t = r/\theta_t$. For values of $r_t \leq \underline{r}_t$, the model generates a "liquidity trap" as banks are no longer interested to use the available liquidity to lend to …rms but, instead, will invest it in the safe asset.

3.4. Central Bank

The central bank (CB) has a monopoly on issuing "loanable funds", or "money". Banks discount their loans to …rms with the CB and, in exchange, receive money, M_t, which they can lend to …rms.

Central Bank's Balance Sheet

Securities from Banks B_t^0	Reserves (Deposits) from Banks R_t
Safe Assets $(\eta_t^0 - \eta_t)p_t$	Net Worth E_t

[3] This is related, part, to the fact that we do not have a demand for money for transaction purposes in our model and, part, to the fact that, in a more complete model, what is actually …xed is the lower bound $\underline{\gamma}$, while the actual reserve ratio should be treated as an equilibrium outcome.

Loanable funds are generated, first of all, by the CB by purchasing safe assets from banks during open market operations. Secondly, loanable funds are also created by the CB by discounting securities banks receive from the firms they are financing and increasing banks' reserves in exchange. Banks' reserves with the CB can be swapped at any time with currency and are, in the simplified case we are studying here, the only amount of fiat money in existence.

3.5. Market clearing

1. Goods market:

$$c_t + \int_0^1 k_{t+1}(\alpha)d\alpha + \eta^0_{t+1} = \int_0^1 y_t(\alpha)d\alpha + (1+r)\eta_t$$

2. Stock market:

$$s_t(\alpha) = 1, \text{ for all } \alpha$$

3. Loan market:

$$B_t + D_t + R_t = d_t + m_t + e_t$$

4. Labor market:

$$B_t / w_t = \ell_t$$

Walras' Law and the household's budget contraint implying that the market for bank's deposits will also clear when these four equations are satisfied.

Now, if we knew how to do it, we would move on and characterize the properties of the competitive equilibrium of this economy. Because we do not, we try a round-about way, i.e. we try to figure out if there is, at least, some constrained central planner problem that may help characterizing the equilibrium of the decentralized economy when the Central Bank follows an optimal policy. The, more interesting, case of general or sub-optimal Central Bank's policies will have to wait for future versions of these notes.

4. Competitive Equilibrium

Let us go back to the decentralized economy described above and consider equilibrium under two different institutional settings: with banks but without the CB, and with both banks and the CB. The decentralization of the frictionless Brock&Mirman-like allocation is well known and will be omitted.

4.1. Banks but not Central Bank

In this setting, bank notes must be "backed" by some "gold standard like" mechanism, i.e. the banks will issue credit to the firms and back-up their payment promises with (some of) the real resources η^0_t deposited with them by the households. Because of this, the equilibrium (and optimal) bank multiplier is finite in this setting. Only when fiat money and a central bank are introduced we can dispense with this restriction.

The conditions under which such bank lending takes place and the amounts involved, are determined by the same …rst order conditions listed above for the case in which the central planner could only use inside and not outside money.

The price level is stationary and countercyclical as it is pinned-down by the marginal utility of consumption. The real wage may be either as it depends on the relative magnitudes of the technology shock, the curvature of the aggregate technology and that of the disutility of labor.

To be completed.

4.2. Banks and Central Bank

In this setting there is both inside and outside money. Banks will issue credit to …rms using either the deposits as a guarantee or the funds issued by the Central Bank. How much of one and of the other kind of money will be used is determined by the …rst order conditions solving the banks' pro…t maximization problem presented in Section 2. It can be seen that this depends on the technology shock, the productivityof the safe technology, the interest rate r_t the CB chooses and the leveraging ratio it imposes on banks.

What's more important, though, is that because the CB can determine the opportunity cost of holding funds with it, as reserves, or of borrowing funds from it, at the discount window, it will also determine the price at which banks may or may not be willing to lend to the private sectors the deposits it receives by the the private sector itself via the banking multipliers. The leverage ratio

The nominal wage increases while the real wage is constant.

As the nominal value of shares increases, so do its real valuation until full employment is reached. After that only the nominal value increases. The share of GDP accruing to capital may go either way.

2 For the **general case** in which $v(\ell)$ is strictly convex, the analysis is more complex and I do not yet have a set of definitive results to present. The complication comes from the following fact: because the CB fixes both a nominal interest rate and a leverage ratio, when the latter is particularly low the marginal productivity of labor may still be higher than the marginal disutility of labor (at equilibrium). Still if all the inside money that is profitable to lend has been lent (assume z is high so it is not convenient to shift resources from the linear to the concave technology) and the banks have reached the maximum leverage ratio allowed, additional lending to hire more workers becomes impossible. I conjecture that in this case we would observe a reduction in the nominal price level, with respect to the case in which the optimal quantity of outside money is issued, and the allocation would be inefficient, with too little output and too little employment. Hence, an increase in the quantity of money, due for example to an increase in the maximum allowed leverage ratio, would increase efficiency and output, together with the price level It is also unclear, yet, if the opposite is the case when the monetary stance is very lax and the quantity of outside money is, in equilibrium higher than the optimal one. That this will generate an increase in the nominal values of assets and in the general price level (i.e. "inflation") seems clear enough, but it is not clear if this can also lead to "too high" a level of employment.

In either case, monetary policy has real effects in this particular kind of world; moreover it may cause an increase in the valuation of assets in the presence of an otherwise stable price level.

To be completed.

5. Long and Short Term: Taking a Lunch Break

One of the main tenet of the class of theories I am investigating is that aggregate models are good for nothing, as their very special properties obscure the key sectorial forces at work in real economies. In particular, the Austrian theorists say, one must recognize that some production processes are more capital intensive than other, hence takes longer (are more "time intensive" or have a "longer production period") to connect the dots going from inputs to output. It is indeed

In particular, the Austrian main argument, stressing the relevance of a multisector disaggregated analysis, is summarized in the following way on the same page on Wikipedia quoted earlier:

According to the theory, the boom-bust cycle of malinvestment is generated by excessive and unsustainable credit expansion to businesses and individual borrowers by the banks.[18] This credit creation makes it appear as if the supply of "saved funds" ready for investment has increased, for the effect is the same: the supply of funds for investment purposes increases, and the interest rate is lowered.[19] Borrowers, in short, are misled by the bank inflation into believing that the supply of saved funds (the pool of "deferred" funds ready to be invested) is greater than it really is. When the pool of "saved funds" increases, entrepreneurs invest in "longer process of production," i.e., the capital structure is lengthened, especially in the "higher orders", most remote from the consumer. Borrowers take their newly acquired funds and bid up the prices of capital and other producers' goods, which, in the theory, stimulates a shift of investment from consumer goods to capital goods industries. Austrians further contend that such a shift is unsustainable and must reverse itself in due course. [...]

The proportion of consumption to saving or investment is determined by people's time preferences, which is the degree to which they prefer present to future satisfactions. [...]

Because the debasement of the means of exchange is universal, many entrepreneurs can make the same mistake at the same time (i.e. many believe investment funds are really available for long term projects when in fact the pool of available funds has come from credit creation - not real savings out of the existing money supply). As they are all competing for the same pool of capital and market share, some entrepreneurs begin to borrow simply to avoid being "overrun" by other entrepreneurs who may take advantage of the lower interest rates to invest in more up-to-date capital infrastructure. A tendency towards over-investment and speculative borrowing in this "artificial" low interest rate environment is therefore almost inevitable.[18]

This new money then percolates downward from the business borrowers to the factors of production: to the landowners and capital owners who sold assets to the newly indebted entrepreneurs, and then to the other factors of production in wages, rent, and interest. Austrian economists conclude that, since time preferences have not changed, people will rush to reestablish the old proportions, and demand will shift back from the higher to the lower orders. In other words, depositors will tend to remove cash from the banking system and spend it (not save it), banks will then ask their borrowers for payment and interest rates and credit conditions will deteriorate.[18]

Austrian economists theorize that capital goods industries will find that their investments have been in error; that what they thought profitable really fails for lack of demand by their entrepreneurial customers. Higher orders of production will have turned out to be wasteful, and the malinvestment must be liquidated.[22] In other words, the particular types of investments made during the monetary boom were inappropriate and "wrong" from the perspective of the long-term financial sustainability of the market because the price signals stimulating the investment were distorted by fractional reserve banking's recursive lending "ballooning" the pricing structure in various capital markets.

This sounds pretty complicated - at least to me - and I bear no hope to have fully understood all its subtleties and nuances, let alone being able to provide a formal representation of all the ideas packed in the large literature supporting the long citation I just forced you to read. Nevertheless, I will try making a first step in the direction Austrian theorists indicate we should travel. Other will follow in future versions of this work or, possibly, through the work of more able researchers.

5.1. Longer and shorter production processes

Because we need at least two technologies, each with a di¤erent time length between inputs and outputs, I will assume that one of my production processes takes longer than the other to complete. Because, at least according to the theories I am trying to formalize, the longer production processes are those in which investment, …nanced by borrowing from banks, mostly takes place, I assume the risky production process takes longer to complete than the safe one.

Therefore, we split period t in two subperiods, before (t^{am}) and after (t^{pm}) lunch. In this new world, the day starts in the morning (like in every other world) and inputs are assigned to both the risky and the AM-safe production process. In other words, the …rst production sub-period begins in the morning, when and η_t^{am} is applied, and ends right before lunch, when Y_t^{am} becomes available.

$$Y_t^{am} = (1+r)\eta_t^{am}$$

The output available at lunch time can either be eaten or reivensted in the PM-safe technology

$$Y_t^{am} \geq c_t^{am} + \eta_t^{pm},$$

which yelds its output in the evening

$$Y_t^{pm} = (1+r)\eta_t^{pm}$$

when also the risky production process ends

$$Y_t^1 = z_t F(k_t, \ell_t) = \quad A_t(\alpha)k_t(\alpha)\ell_t(\alpha)^{1\ \#} d\alpha;\ k_t = \quad k_t(\alpha)d\alpha,\ \ \ell_t = \quad \ell_t(\alpha)d\alpha.$$

During the evening, total output is split as in the previous world, i.e.

$$Y_t^1 + Y_t^{pm} \geq c_t^{pm} + \eta_{t+1}^0 + k_{t+1}$$

5.2. Preferences

De…ne a new intratemporal utility function as

$$U(c_t^{am}, c_t^{pm}) = u(c_t^{am}) + \beta u(c_t^{pm});\ \ \beta > 0.$$

Intertemporal utility is now

$$E_0 \sum_{t=0}^{1} \delta^t [U(c_t^{am}, c_t^{pm}) + v(L - \ell_t)].$$

5.3. First best

The …rst best allocation is rather simple, insofar as it mimics the one for the standard Brock&Mirman model, with the added complexity that, because $\eta_{t+1}^{am} \leq \eta_{t+1}^0$ we must have

$$u'(c_t^{pm}) = \delta(1+r)u'\left(c_{t+1}^{am}\right),$$

and because $Y_{t+1}^{am} \geq c_{t+1}^{am} + \eta_{t+1}^{pm}$, we also have

$$u'\left(c_{t+1}^{am}\right) = \beta(1+r)u(c_{t+1}^{pm}).$$

On the other hand, because $Y_t^1 + Y_t^{pm} \geq c_t^{pm} + \eta_{t+1}^0 + k_{t+1}$,

$$u'(c_t^{pm}) = \delta\beta E_t\left[u'\left(c_{t+1}^{pm}\right)z_{t+1}F_1(k_{t+1}, \ell_{t+1})\right].$$

Finally, because $Y_t^1 = z_t F(k_t, \ell_t)$, the e¢ cient allocation of labor is

$$z_t F_2(k_t, \ell_t) = v'(L - \ell_t)/u'(c_t^{pm}).$$

5.4. Market Structure and Budget Constraints

A number of alternative market structures are worth considering. [What follows is a cursory summary of tentative results and more or less well proved conjectures.] **To be completed.**

5.4.1. Private segmented banks

Assume only private banks exist, which are of two kinds: those making long term loans to …rms operating the risky technology (IBs), and those taking short term deposits to invest in the safe technology (CBs). In our simple setting, with no uncertainty and a representative household, the CBs are just production …rms running the AM-safe and PM-safe technologies on behalf of households.

The IBs, on the other hand, would have to take in long-term deposits from households to back their issuance of the bank-notes necessary to provide the risky …rms with the credit they need. The amount of such deposits will depend on the equilibrium reserve ratio the IBs manage to use: the lower the reserve ratio the lower the amount of deposits taken in. The relevant point, obviously, is that (contrary to a classical Austrian claim) the lower is the fraction of deposits held on reserve, the more e¢ cient is the allocation.

5.4.3. Private segmented banks and a Central Bank

Add a Central Bank to the stylized Glass and Steagall world described above. If the Central Bank follows the optimal policy, the …rst best can be achieved again. The optimal policy consists in issuing …at money to lend to the IBs, in exchange for the bonds issued by the risky …rms being …nanced, while the Central Bank can safely ignore the CBs activities. Note that, in this setting, the optimal policy requires the Central Bank to set the "long-term" rate to zero, where "long-term" here is de…ned by the length of the risky production period.

To the extent the two kinds of banks remain separated, a Central Bank that makes "mistakes" (e.g. by moving around the interest rate in an unpredictable fashion) may cause under-

The main conclusions we derive from our theoretical analysis are the following:

As the Central Bank increases the nominal amount of funds, M_t, available, and relaxes the leverage constraint:

- *Output in the home country increases together with the market value (net worth) of firms.*
- *The nominal price of output remains constant, or grows at the same rate at which the price of output in the foreign country grows when expressed in the home currency.*
- *The nominal (and real, therefore) wage is constant.*
- *As the nominal value of shares increases, so do its real valuation and the share of GDP accruing to capital.*

In other words, monetary policy has real effects in this particular kind of world.

Conjecture: in the model with a non-tradeable good (produced by some non-tradeable asset) the nominal (hence, real) value of such asset also increases, roughly one-to-one with the stock market valuation of the risky technology.

To be completed.

7. Conclusions

To be added

8. References

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